|  |
| --- |
| Specialist Mathematics 2025 v1.2  Subject matter resource |

The following resource provides the subject matter from the Specialist Mathematics syllabus, formatted in Microsoft Word. The mathematical equations have been created using Equation Editor, which allows teachers to copy and paste them into teaching, learning and assessments resources.

This document is intended to support teachers by offering a convenient reference to the syllabus content. This document may not capture further amendments and it does not replace the syllabus as the authoritative source. Teachers should verify details by consulting the syllabus directly.

### Unit 1: Combinatorics, proof, vectors and matrices

#### Topic 1: Combinatorics

##### Sub-topic: Introduction to counting techniques (4 hours)

* Use the inclusion-exclusion principle formulas to determine the number of elements in the union of two and the union of three sets.
* Use the multiplication principle.
* Use the addition principle.

##### Sub-topic: Permutations (ordered arrangements) and combinations (unordered selections) (8 hours)

* Define and use permutations.
* Use factorial notation.
* Use the notation  to represent the number of ways of selecting objects from distinct objects where order is important.
* Solve problems that involve permutations.
* Solve problems that involve permutations with restrictions including repeated objects, specific objects grouped together and selection from multiple groups.
* Define and use combinations.
* Use the notation and to represent the number of ways of selecting objects from distinct objects where order is not important.
* Solve problems that involve combinations.
* Solve problems that involve combinations with restrictions including specific objects grouped together and selection from multiple groups.
* Model and solve problems that involve permutations and combinations including probability problems, with and without technology.

#### Topic 2: Introduction to proof

##### Sub-topic: The nature of proof (5 hours)

* Use implication, converse, equivalence, negation, contrapositive.
* Use proof by contradiction.
* Use the symbols for implication ( ⇒ ), equivalence ( ⟺ ), and equality ( ).
* Use the quantifiers ‘for all’ and ‘there exists’ .
* Define and use set notation of number systems, including integers (), positive integers (), negative integers (), rational numbers (), irrational numbers (), and real numbers ().
* Use the set notation symbol ‘is an element of’ .
* Use examples and counterexamples.

##### Sub-topic: Rational and irrational numbers (5 hours)

* Prove results involving integers, e.g. proving that the product of two consecutive odd numbers is an odd number and is an even number.
* Express rational numbers as terminating or eventually recurring decimals and vice versa.
* Prove irrationality by contradiction.

#### Topic 3: Vectors in the plane

##### Sub-topic: Representing vectors in the plane by directed line segments (5 hours)

* Examine examples of vectors including displacement, velocity and force.
* Understand the difference between a scalar and a vector including distance and displacement, speed and velocity, and magnitude of force and force.
* Define and use the magnitude and direction of a vector.
* Understand and use vector notation: and unit vector notation .
* Understand and use vector equality.
* Represent and use a scalar multiple of a vector.
* Use the triangle rule to represent the resultant vector from the sum and difference of two vectors.
* Represent a vector in the plane using a combination of the sum, difference and scalar multiple of other vectors.

##### Sub-topic: Vectors in two dimensions (5 hours)

* Use ordered pair notation and column vector notation to represent a position vector in two dimensions.
* Calculate the magnitude and direction of a vector.
* Calculate and use a unit vector, , in the plane.
* Define and use unit vectors and the perpendicular unit vectors and .
* Express a vector in Cartesian (component) form using the unit vectors and .
* Understand and express a vector in the plane in polar form using the notation .
* Convert between Cartesian form and polar form, with and without technology.
* Understand and use the Cartesian form and polar form of a vector.

#### Topic 4: Algebra of vectors in two dimensions

##### Sub-topic: Algebra of vectors in two dimensions (12 hours)

* Examine and use addition and subtraction of vectors in Cartesian form.
* Define and use multiplication by a scalar of a vector in Cartesian form.
* Determine a vector between two points.
* Define and use a vector representing a section of a line segment, including the midpoint of a line segment.
* Define and use the scalar (dot) product.
* Examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular.
* Define and use scalar and vector projections of vectors.
  + - scalar projection of on :
    - vector projection of on :
* Apply the scalar product to vectors expressed in Cartesian form.
* Resolve vectors into and components.
* Model and solve problems that involve displacement, force, velocity and relative velocity using the above concepts.
* Model and solve problems that involve motion of a body in equilibrium situations, including vector applications related to smooth inclined planes (excluding situations with pulleys and connected bodies).

#### Topic 5: Matrices

##### Sub-topic: Matrix arithmetic and algebra (11 hours)

* Understand the matrix definition and notation.
* Define and use addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity and multiplicative inverse.
* Use matrix algebra properties, including
  + - (commutative law for addition)
    - ***0*** (additive identity)
    - ***0*** (additive inverse)
    - (multiplicative identity)
    - (multiplicative inverse)
    - (left distributive law)
    - (right distributive law)
* Recognise that matrix multiplication in general is not commutative.
* Calculate the determinant and multiplicative inverse of matrices, with and without technology.
  + - If then
* Use matrix algebra to solve matrix equations that involve matrices of up to dimension , including those of the form , and , with and without technology.
* Model and solve problems that involve matrices of up to dimension , including the solution of systems of linear equations, with and without technology.

### Unit 2: Complex numbers, further proof, trigonometry, functions and transformations

#### Topic 1: Complex numbers

##### Sub-topic: Introduction to complex numbers (4 hours)

* Define the imaginary number as a root (solution) of the equation .
* Define and use set notation of the number system for complex numbers ().
* Use complex numbers in the form where and are the real and imaginary parts (components) and of a complex number .
* Determine and use complex conjugates.
* Perform complex-number arithmetic: addition, subtraction, multiplication and division, with and without technology.

##### Sub-topic: The complex plane (the Argand plane) (6 hours)

* Sketch and use complex numbers as points in the complex plane with real and imaginary parts as Cartesian coordinates.
* Understand and use addition of complex numbers as vector addition in the complex plane.
* Understand and use location of complex conjugates in the complex plane.
* Understand and use multiplication by a complex number as a linear transformation in the complex plane.

#### Topic 2: Complex arithmetic and algebra

##### Sub-topic: Complex arithmetic using polar form (4 hours)

* Use the modulus of a complex number and the principal argument of a non-zero complex number .
* Understand the difference between the argument, , and the principal argument, of a non-zero complex number .

* Express a complex number in Cartesian form and polar form.
  + - or
* Convert between Cartesian form and polar form.
* Understand and use multiplication, division of complex numbers in polar form and the geometric interpretation of these.
* Sketch and use complex numbers in polar form as polar coordinates.

##### Sub-topic: Subsets of the complex plane (the Argand plane) (3 hours)

* Identify and sketch subsets of the complex plane determined by straight lines and circles, e.g. , , and .

##### Sub-topic: Roots of real quadratic equations (4 hours)

* Determine complex conjugate solutions of real quadratic equations with real coefficients using factorisation, completing the square and the quadratic formula, with and without technology.
* Determine and use linear factors of quadratic polynomials with real coefficients that involve the complex conjugate root theorem, e.g. determine the coefficients of a real quadratic equation given one complex root.

#### Topic 3: Circle and geometric proofs

##### Sub-topic: Circle properties and their proofs (6 hours)

* Prove the circle properties
  + the angle at the centre subtended by an arc of a circle is twice the angle at the circumference subtended by the same arc
  + an angle in a semicircle is a right angle
  + angles at the circumference of a circle subtended by the same arc are equal
  + the alternate segment theorem
  + the opposite angles of a cyclic quadrilateral are supplementary and its converse
  + a tangent drawn to a circle is perpendicular to the radius at the point of contact and its converse.
* Solve problems finding unknown angles and lengths and prove further results using the circle properties listed above.

##### Sub-topic: Geometric proofs using vectors (6 hours)

* Prove the diagonals of a parallelogram meet at right angles if and only if it is a rhombus.
* Prove midpoints of the sides of a quadrilateral join to form a parallelogram.
* Prove the sum of the squares of the lengths of a parallelogram’s diagonals is equal to the sum of the squares of the lengths of the sides.
* Prove an angle in a semicircle is a right angle.

#### Topic 4: Trigonometry and functions

##### Sub-topic: Sketching graphs (3 hours)

* Use and apply the notation for the absolute value for the real number and the graph of .
* Understand and use the relationship between the graph of and the graphs of   
   , and .

##### Sub-topic: The reciprocal trigonometric functions, secant, cosecant and cotangent (3 hours)

* Define and use the reciprocal trigonometric functions to determine their simplified exact values and sketch their graphs.

##### Sub-topic: Trigonometric identities (6 hours)

* Prove and apply the Pythagorean identities.
* Prove and apply the angle sum, difference and double-angle identities for sines and cosines.

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

* Prove and apply the identities for products of sines and cosines expressed as sums and differences.
* Convert sums to or and apply these to sketch graphs.
* Model and solve problems that involve equations of the form .
* Prove and apply multi-angle trigonometric identities up to angles of using the identities listed above, e.g. and

#### Topic 5: Matrices and transformations

##### Sub-topic: Transformations in the plane (10 hours)

* Understand translations and their representation as column vectors.
* Use basic linear transformations: dilations of the form , rotations about the origin and reflection in a line that passes through the origin, and the representations of these transformations by matrices.
  + - dilation of factor parallel to the -axis and factor parallel to the -axis:
    - rotation of angle anticlockwise about the origin:
    - reflection in the line :
* Apply these transformations to points in the plane and polygons.
* Understand and use composition of linear transformations and the corresponding matrix products.
* Understand and use inverses of linear transformations and the relationship with the matrix inverse.
* Understand and use the relationship between the determinant and the effect of a linear transformation on area.
* Determine geometric results by matrix multiplications, e.g. showing that the combined effect of two reflections in lines through the origin is a rotation.

### Unit 3: Further complex numbers, proof, vectors and matrices

#### Topic 1: Further complex numbers

##### Sub-topic: Complex arithmetic using polar form (3 hours)

* Prove complex number identities involving modulus and argument, e.g.   
  , and .
* Use De Moivre’s theorem for integral powers.

##### Sub-topic: Roots of complex numbers (3 hours)

* Determine and examine the th roots of unity and their location on the unit circle.
* Determine and examine the th roots of complex numbers and their location in the complex plane.

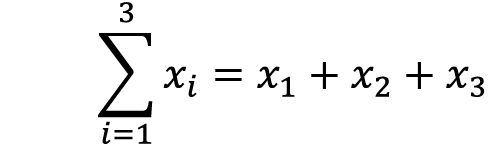
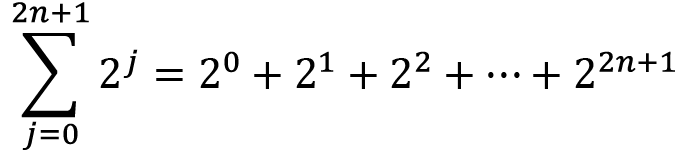
##### Sub-topic: Factorisation of polynomials (5 hours)

* Apply the factor theorem and the remainder theorem for polynomials.
* Understand and use the complex conjugate root theorem for polynomials with real coefficients, e.g. factorise a cubic polynomial with real coefficients given one factor.
* Solve polynomial equations over to order 4 including those with real and imaginary coefficients, e.g. solve and .

#### Topic 2: Mathematical induction and trigonometric proofs

##### Sub-topic: Mathematical induction (8 hours)

* Understand the nature of inductive proof including the use of initial statement, assumption statement, inductive step and conclusion.
* Use sigma notation to represent a sum, e.g.

and 

* Prove results for sums for any positive integer .
* Prove divisibility results for any positive integer .
* Prove De Moivre’s theorem for powers of positive integers.

##### Sub-topic: Trigonometric proofs using De Moivre’s theorem (2 hours)

* Prove multi-angle trigonometric identities up to angles of by equating parts using the binomial expansion and De Moivre’s theorem, e.g. and .

#### Topic 3: Vectors in two and three dimensions

##### Sub-topic: Vectors in three dimensions (3 hours)

* Use Cartesian coordinates for three-dimensional space, including plotting points.
* Use ordered triple notation and column vector notation to represent a position vector in three dimensions.
* Calculate the magnitude of a vector
* Calculate and use a unit vector, , in three-dimensional space.
* Define and use unit vectors and the perpendicular unit vectors , and .
* Express a vector in Cartesian (component) form using the unit vectors , and .
* Define and use the altitude angle .

##### Sub-topic: Algebra of vectors in three dimensions (3 hours)

* Examine and use addition and subtraction of vectors in Cartesian form.
* Use multiplication by a scalar of a vector in Cartesian form.
* Determine a vector between two points.
* Use a vector representing a section of a line segment, including the midpoint of a line segment.
* Use the scalar (dot) product.
* Examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular.
* Use scalar and vector projections of vectors.
  + - scalar projection of on :
    - vector projection of on :
* Apply the scalar product to vectors expressed in Cartesian form.
* Model and solve problems that involve displacement, force, velocity and relative velocity using the above concepts.
* Use vectors to prove geometric results in two dimensions (other than those listed in Unit 2 Topic 3) and in three dimensions.

##### Sub-topic: Vector and Cartesian equations (7 hours)

* Understand and use equations of spheres.
  + - equation of sphere:
* Use vector equations of curves in two or three dimensions involving a parameter, and determine a ‘corresponding’ Cartesian equation in the two-dimensional case.
* Determine vector, parametric and Cartesian equations of straight lines and straight-line segments given the position of two points, or equivalent information, in both two and three dimensions.
  + - vector equation of line:
    - parametric equations of line:
    - Cartesian equation of line:
* Define and use the vector (cross) product to determine a vector normal to a given plane, with and without technology.
* Use vector methods in applications, including areas of shapes and determining vector and Cartesian equations of a plane and of regions in a plane.
  + - vector equation of plane:
    - Cartesian equation of plane:

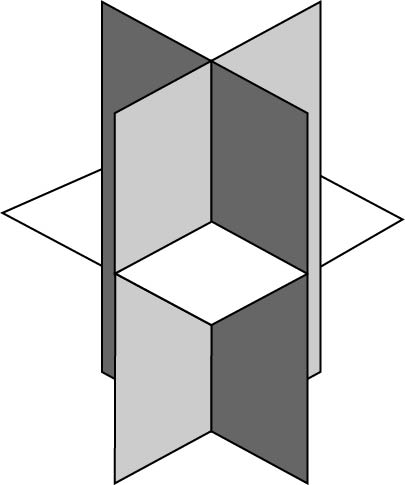
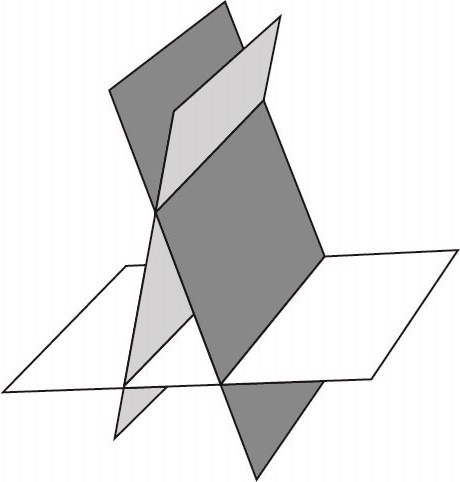
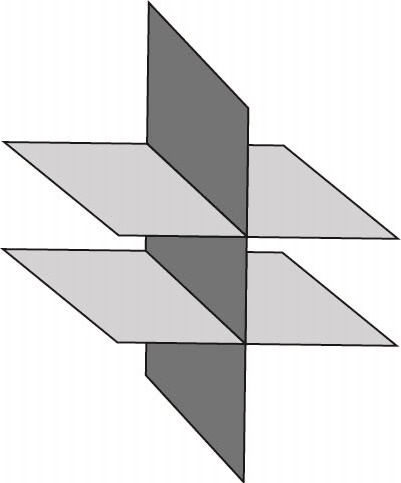
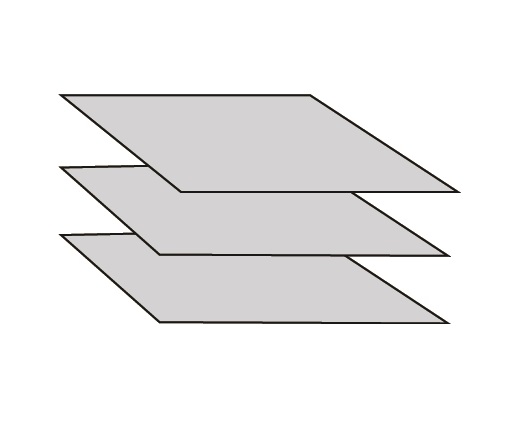
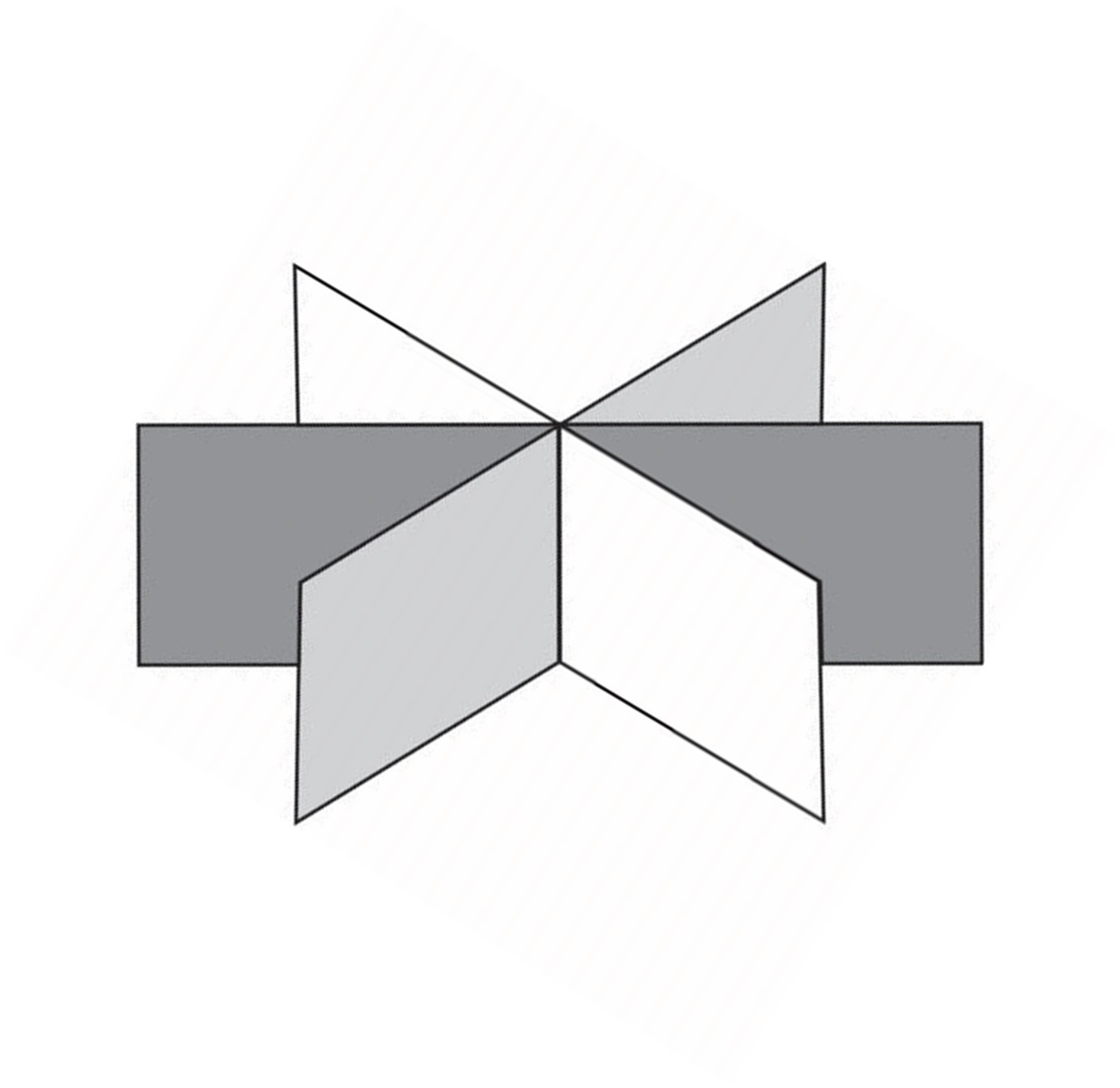
#### Topic 4: Vector calculus

##### Sub-topic: Vector calculus (9 hours)

* Understand and use position of vectors as a function of time.
* Understand and use the Cartesian equation of a path given as a vector equation in two dimensions, including circles, ellipses and hyperbolas.
  + - equation of circle:
    - equation of ellipse:
    - equation of hyperbola: or
* Understand and use the position of two particles, each described as a vector function of time, and determine if their paths cross or if the particles meet.
* Differentiate and integrate a vector function with respect to time.
* Use vector calculus to determine equations of motion of a particle travelling in a straight line with both constant and variable acceleration.
* Apply vector calculus to model and solve problems that involve motion in a plane, including projectile and circular motion, with and without technology.

#### Topic 5: Further matrices

##### Sub-topic: Matrix algebra and systems of equations (5 hours)

* Calculate the determinant and multiplicative inverse of square matrices of any order, with technology.
* Use the determinant to determine whether a square matrix of any order is singular or non-singular.
* Use matrix algebra to solve matrix equations that involve matrices of beyond dimension , including those of the form , and , with technology.
* Model and solve problems that involve matrices of beyond dimension , including the solution of systems of linear equations, with technology.
* Recognise the general form of a system of linear equations in several variables and use Gaussian techniques of elimination on an augmented matrix to solve a system of linear equations, with and without technology.
* Examine the three cases for solutions of systems of equations — a unique solution, no solution and infinitely many solutions — and the geometric interpretation of a solution of a system of equations with three variables including
  + - a unique solution  
      
    - no solution   
        
    - infinitely many solutions  
      

##### Sub-topic: Applications of matrices (7 hours)

* Model and solve problems that involve real-life situations using matrices, including Dominance and Leslie matrices.
* Investigate how matrices have been applied in other real-life situations, e.g. Leontief, Markov, area, cryptology, eigenvectors and eigenvalues.

**Note:** The external examination may assess only Dominance and Leslie matrices.

### Unit 4: Further calculus and statistical inference

#### Topic 1: Integration techniques

##### Sub-topic: Integration techniques (10 hours)

* Integrate using the trigonometric identities ,   
  , and .
* Establish and use the formula
* Use substitution to integrate expressions of the form .
* Establish and use the formula for and   
  for .
* Understand and use the inverse trigonometric functions: arcsine, arccosine and arctangent.
* Use the derivative of the inverse trigonometric functions: arcsine, arccosine and arctangent.
* Integrate expressions of the form and .
* Use partial fractions for integration involving two distinct linear factors in the denominator, e.g. .
* Integrate by parts.

#### Topic 2: Applications of integral calculus

##### Sub-topic: Applications of integral calculus (10 hours)

* Apply techniques from Unit 4 Topic 1 Sub-topic: Integration techniques to calculate areas between curves determined by functions, with and without technology.
* Determine volumes of solids of revolution about either axis, with and without technology.
  + - about the -axis:
    - about the -axis:
* Use Simpson’s rule to approximate an area and the value of a definite integral, with and without technology.

* + - where
* Understand and use the probability density function, for , of the exponential random variable with parameter .
  + - mean:
    - standard deviation:
* Model and solve problems that involve exponential random variables and associated probabilities and quantiles, with and without technology.

#### Topic 3: Rates of change and differential equations

##### Sub-topic: **Rates** of change (3 hours)

* Use implicit differentiation to determine the gradient of curves whose equations are given in implicit form.
* Model and solve related rates problems as instances of the chain rule including situations that involve surface area and volume of cones, pyramids and spheres, with and without technology.

##### Sub-topic: Differential equations (8 hours)

* Determine general and particular solutions of first-order differential equations of the form , differential equations of the form and differential equations of the form  using separation of variables.
* Understand and use slope (direction or gradient) fields of a first-order differential equation.
* Model and solve problems using provided differential equations, including the logistic equation, Newton’s law of cooling and radioactive decay, with and without technology.

#### Topic 4: Modelling motion

##### Sub-topic: Modelling motion (11 hours)

* Understand and use momentum, constant force, non-constant force, resultant force, action and reaction.
* Understand and use motion of a body in non-equilibrium situations under concurrent forces.
* Understand and use the expressions , , and to represent the acceleration of an object moving in a straight line.
* Model and solve problems that involve motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion, vertical motion under gravity with and without air resistance, and motion of a body in non-equilibrium situations on a smooth inclined plane (excluding situations with pulleys and connected bodies).
  + - If then or

#### Topic 5: Statistical inference

##### Sub-topic: Sample means (7 hours)

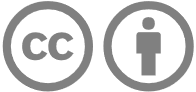
* Understand the concept of the sample mean as a random variable whose value varies between samples where is a random variable with mean and the standard deviation .
* Use repeated random sampling data from a variety of distributions and a range of sample sizes to examine properties of the distribution of across samples of a fixed size, including its mean , its standard deviation (where and are the mean and standard deviation   
  of ) and its approximate normality if is large.
* Recognise and use the link between the normal distribution of the sample mean and the statistical notation .
* Use repeated random sampling data from a variety of distributions and a range of sample sizes to examine the approximate standard normality of for large samples (), where is the sample standard deviation (Central limit theorem).
* Model and solve problems that involve sample means, with and without technology.

##### Sub-topic: Confidence intervals for means (6 hours)

* Understand the concept of an interval estimate for a parameter associated with a random variable.
* Understand and use the approximate confidence interval , as an interval estimate for , the population mean, where is the appropriate quantile for the standard normal distribution.
* Understand and use the approximate margin of error.
* Understand and use the relationship between margin of error, level of confidence and sample size.
* Understand and use the concept that there are variations in confidence intervals between samples and that most but not all confidence intervals contain .
* Use and to estimate and , to obtain approximate intervals covering desired proportions of values of a normal random variable and compare with an approximate confidence interval for .
* Model and solve problems that involve interval estimates for sample means, with and without technology.

## More information

If you would like more information, please email the Mathematics learning area branch at [maths@qcaa.qld.edu.au](mailto:maths@qcaa.qld.edu.au).

[](https://www.qcaa.qld.edu.au/copyright) © State of Queensland (QCAA) 2024

**Licence:** <https://creativecommons.org/licenses/by/4.0> **| Copyright notice:** [www.qcaa.qld.edu.au/copyright](https://www.qcaa.qld.edu.au/copyright) —   
lists the full terms and conditions, which specify certain exceptions to the licence. **|   
Attribution** (include the link): © State of Queensland ([QCAA](https://www.qcaa.qld.edu.au/copyright)) 2024 [www.qcaa.qld.edu.au/copyright](https://www.qcaa.qld.edu.au/copyright).